USE OF THE GROVER NOMOGRAM FOR THE APPROXIMATE CALCULATION OF THE TEMPERATURE OF A THERMALLY INSULATED CYLINDRICAL WALL

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It is proposed to use the temperature nomogram constructed for a flat two-layer (metal-insulation) wall to determine the temperature of a cylindrical wall. Formulas for the correction factors are presented together with examples of wall temperature calculations for various combinations of the thermal parameters.

Nomograms for the rapid estimation of the temperatures of bodies of various shapes (plates, cylinders, spheres) are presented in [1–3]. In 1957 Grover and Holter [4] proposed a nomogram for calculating the temperature of a flat metal wall with unilateral heat exchange with the medium through a layer of thermal insulation. The nomogram is very convenient since for a known value of the Fourier number it makes it possible to determine the unknown wall temperature by interpolation with respect to a single generalized parameter μ. This parameter is a combination of the Biot number, calculated for the insulation, and the ratio R of the total specific heats of the metal and the insulation:

$$\mu = k + \frac{1}{\mathrm{Bi}} + \frac{k}{\mathrm{Bi}}. \tag{1}$$

Calculations show that this nomogram can also be used to determine the temperature of a cylindrical wall, if one substitutes for the parameter μ the parameter μ^*:

$$\mu^* = k^* + \frac{1}{\mathrm{Bi}^*} + \frac{k^*}{\mathrm{Bi}^*}, \tag{2}$$

where

$$k^* = \Delta k + k f_1; \tag{3}$$

$$\mathrm{Bi}^* = \mathrm{Bi} f_2. \tag{4}$$

In this case, the quantities k and Bi are computed as for a flat wall. The correction Δk and the coefficients f_1 and f_2 have a simple physical meaning.

Thus, Δk is due to the nonsymmetrical (relative to the middle surface) distribution of the mass of the insulation in the cylindrical wall. The coefficient f_1 takes into account the change in the total specific heat of the metal in the cylindrical wall in comparison with a flat one, and the coefficient f_2 reflects the change in the heat-transfer conditions associated with the curvature of the wall. The wall curvature is conveniently taken into account by introducing the dimensionless parameter δ/r, where δ is the thickness of the layer of insulation (for a homogeneous wall, δ is the wall thickness), r is the radius of the wall surface through which heat exchange with the medium is effected.

The quantity δ/r is assumed to be positive, if the radius increases in a direction away from the heat-transfer surface into the wall.

In the range of variation of the parameters $\delta/r = 0–2$, $k = 0–20$, the following relations for Δk, f_1, and f_2 ensure an accuracy of temperature determination within the limits of accuracy of the nomogram approximation:

$$\Delta k = \frac{\frac{1}{6}\frac{\delta}{r}}{1 + \frac{1}{3}\frac{\delta}{r}}, \tag{5}$$

$$f_1 = \frac{1 + \frac{\delta}{r}}{1 + 0.5\frac{\delta}{r}}, \tag{6}$$

$$f_2 = \frac{1}{1 + 0.37\frac{\delta}{r}}. \tag{7}$$

The table gives the results of a calculation of the relative wall temperature Θ for various combinations of the parameters δ/r, k, and Bi corresponding to $\mu^* = 0.5$, 5, and 50.

The calculation was made on an M-20 computer with the assumptions made in [4]: heat flux one-dimen-

Table

Relative Wall Temperature at Fo = 1

μ^*	0,5			5			50		
$\frac{\delta}{r}$	0	1	2	0	1	2	0	1	2
k	0	0	0	0	0	0	5	5	5
Bi	2	4.11	6.96	0.2	0.316	0.435	0.1333	0.253	0.358
Θ (exact calcn)	0.3691	0.3690	0.3775	0.8550	0.8512	0.8491	0.9880	0.9874	0.9876
k	0.5	0.281	0.2	5	3.65	3.2	20	20	20
Bi	∞	∞	∞	∞	∞	∞	0.7	1.635	2.74
Θ (exact calcn)	0.3701	0.3781	0.3930	0.8551	0.8582	0.8648	0.9870	0.9863	0.9864
Θ (nomogram [43])		0.37			0.85			0.98	

sional; no heat transfer between the medium and the metal surface; thermal resistance of the metal equal to zero.

In conclusion, we note that the Grover nomogram method permits the determination of the wall temperature in the case of negative values of the parameter δ/r and, in particular, for combinations of δ/r, k, and Bi corresponding to negative values of μ^*. The minimum value $\mu^* = -0.25$ corresponds to the conditions: $\delta/r = -1$, $k = 0$, $Bi \to \infty$ (heat transfer between a solid homogeneous cylinder and the medium with a boundary condition of the first kind).

For calculations in the region of negative values, the nomogram should be supplemented by the lines $0 \geq \mu^* \geq -0.25$.

NOTATION

Fo and Bi are, respectively, the Fourier and Biot numbers calculated for the thermal insulation as a flat wall; k is the ratio of the total specific heats of the metal and insulation; δ is the thickness of the insulation, or wall thickness, for a homogeneous wall; r is the radius of the wall surface through which heat exchange with the medium is effected; μ is the generalized parameter of a flat wall; μ^* is the generalized parameter of a cylindrical wall; k* is the ratio of the total heat capacities of the metal and insulation reduced to a cylindrical wall; Bi* is the reduced Biot number; Δk is the correction for cylindricity of the insulation; f_1 and f_2 are, respectively, coefficients which take into account the effect of wall curvature on the ratio of the total heat capacities of the metal and insulation, and the effect on heat exchange with the medium; $\Theta = (T - T_c)/(T_0 - T_c)$ is the relative temperature of the surface at which there is no heat exchange with the medium; T_c is the temperature of the medium; T_0 and T are the initial and variable wall temperatures, respectively.

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